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Effect of Reynolds Number on the Structure of Recirculating Flow

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Introduction

IN this work, we apply the random vortex method[1] to compute the unsteady flow over a rearward-facing step at a wide range of Reynolds number. The simulation is restricted to two-dimensions to limit the computational efforts, and, hence, high Reynolds number effects are manifested only by the growth of perturbations into cyclic variations and the formation of small scales. Attention is focused on predicting the effect of Reynolds number on the dynamics and structure of the flowfield in terms of processes of vortex eddy formation and interaction.

The use of vortex elements to represent the vorticity field and follow its dynamics in a numerical simulation is a natural way to overcome the convective nonlinearity of the Navier-Stokes equations without introducing excessive numerical diffusion. Random walk, used to simulate diffusion in the scheme, is complementary to this representation since it introduces the effect of molecular viscosity without impeding any of the attributes of inviscid vortex methods.[2] The scheme is grid-free and self-adaptive, and computational elements conglomerate around large-velocity gradients to provide high resolution without an unbounded increase in computational effort. Since the Reynolds number controls the relative size of diffusive transport with respect to convective transport, it plays the same role in the numerical algorithm as it does in physics: it controls the balance between convection and diffusion. Random walk introduces a wide spectrum of noise that can excite natural flow instabilities during transition without inhibiting their growth. For more discussion on the algorithm, see Ghoniem and Gagnon[3] and Sethian and Ghoniem.[4] The computer code employed is a modified version of MIMOC.[5]

Solution

Computations were performed for a large matrix of numerical parameters for each Reynolds number to check the convergence of the results.[4] Here, we report on results for a single set of parameters, namely, time step $\Delta t = 0.1$, length of vortex sheets $h = 0.4$, circulation of elementary vortices $\Gamma = 0.025$, and computational domain length $X = 10$. Variables are normalized with respect to the appropriate combination of incoming flow velocity U and step heights S. Four Reynolds numbers, defined as $R = US/\nu$, where ν is the kinematic viscosity, are selected: $R = 50$, 125, 500, 5000. For all values of R, the flow reached a stationary state during the first 500 time steps. The data are analyzed for time steps 501-1000.

Vorticity Structure

Instantaneous streamlines are obtained by tracing a set of particles injected at the inlet of the channel and at areas of slow flow until they leave the channel or form closed trajectories. The velocity field is averaged over 10 time steps to remove small noise.

$R = 50$

Figure 1 shows that the recirculation zone is formed of one large eddy, which remains stationary at the corner of the step. The eddy shape can be approximated by an ellipse with the major axis the size of 3S. The recirculation zone maintains a constant size and stays as one structure at all times, indicating limited interactions with the freestream. Thus, convective mixing between the freestream and the recirculating bubble is limited, and all of the exchange takes place by diffusion across the dividing streamline. Stability is clearly observed as small oscillations of the eddy, introduced by the stochastic simulation of diffusion, are damped. The flow can be characterized as viscous and steady.

The computed length of the recirculating zone is 3S, in agreement with the values reported in Denham and Patrick[6] and Armaly et al.[7] Flow visualization studies for a starting flow behind a step without an opposite wall, reported by Honji,[8] show the formation of one eddy at low Reynolds number and confirm its stability.

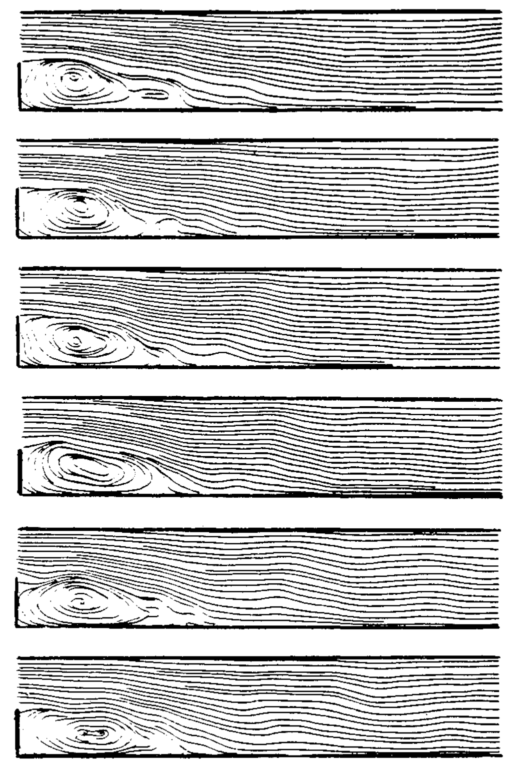

Fig. 1 Instantaneous streamline plots plotted every 10 time steps, starting at $t = 84.8$, $R = 50$.

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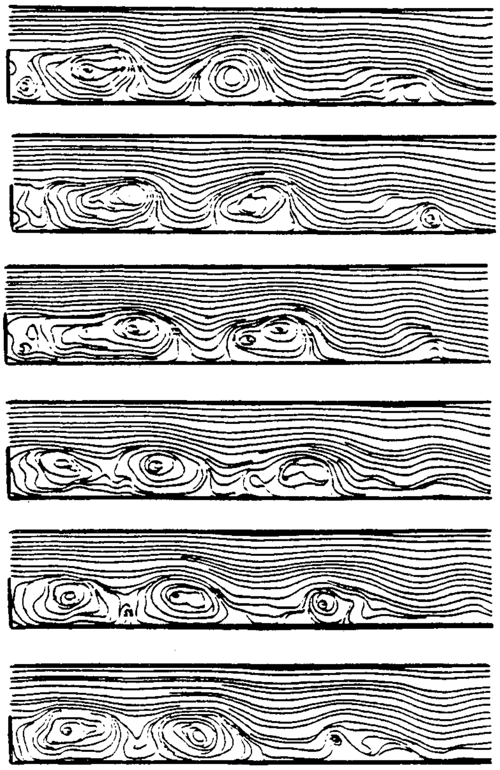

Fig. 2 The breakup of the recirculation eddy at $R = 125$, signifying the establishment of laminar conditions.

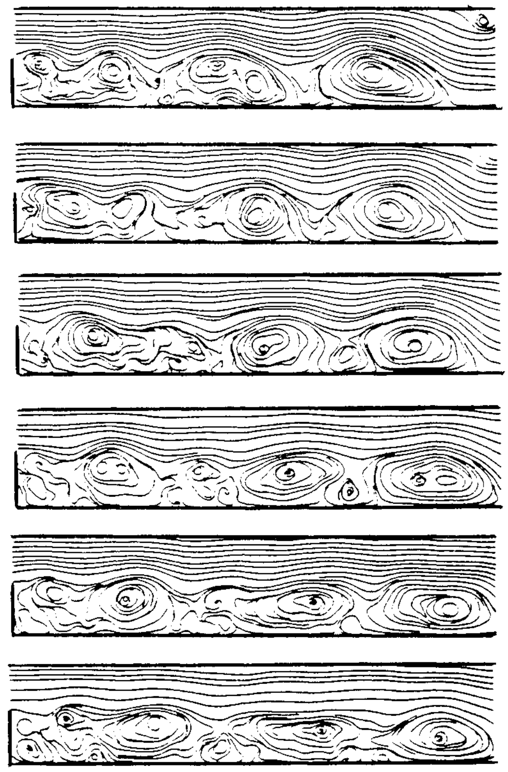

Fig. 3 Instantaneous streamlines for a flow at $R = 500$, plotted every 10 computational time steps, starting at $t = 50.9$.

$R = 125$

As the Reynolds number increases, the recirculation zone grows. Instantaneous streamline plots, depicted in Fig. 2, show the cyclic formation of a large eddy downstream of the step, its detachment from the shear layer, and the subsequent formation of another eddy at the step. The detached eddy moves downsteam at the average flow velocity, and its strength decays before it reaches the end of the computational domain. Concomitant with the formation and separation of large eddies is a low-frequency, small-amplitude oscillation of the average reattachment length.

Honji[8] confirmed that the recirculation eddy disintegrates into two eddies around $R = 120$, followed by the shedding of an eddy at the downstream side. This observation is of particular relevance to our computations, which are unsteady, since we are constantly perturbing the flow with the noise associated with random walk. A more stable flow was obtained when the size of the perturbation was reduced by using smaller vortex elements or shorter time steps.[4] The computed average reattachment length is 6S, which falls well within the experimental results.[6,7] The wide scatter of experimental results can be attributed to the shape of the velocity profile at the step, which depends on the inlet channel length, the upstream contraction, and the flow straightener.

$R = 500$

Figure 3 shows the streamlines of the flow at $R = 500$. At this value, the structure of the recirculation zone changes, and small eddies appear between the corner and the edge of the step. Vortex shedding, which results from a Kelvin-Helmholtz instability of the separating shear layer, is observed. Eddies form at the step, grow by entraining more vortical fluid from the separating shear layer, and move downsteam. As two eddies move downstream, small cross-stream perturbations, coupled with their induced fields, pull their centers closer. Meanwhile, diffusion acts to smear out boundaries between the two eddies. A counter-rotating eddy appears at the corner.

Experimentally, the disintegration of the large eddy into three or more eddies was visualized by Honji[8] for Reynolds numbers over 300. A counter-rotating corner vortex was also described in the same work. Denham and Patrick[6] indicated that the breakup of the original large eddy occurs closer to the step as the Reynolds number increases, with smaller eddies forming at higher values. Transition was also observed in this range of Reynolds number.[9] Wide disagreement concerning when transition starts is reported in the literature of recirculating flows, which cites values for the critical Reynolds number in the 300–2000 range.[6,10] Transition is accompanied by the onset of strong three-dimensional motion, which is not allowed in this numerical study. The apparent similarity in the flow structure is due only to the formation of small scales at the step and their pairing downstream, a process that stems largely from two-dimensional dynamics.

The reattachment length is about 9S (see Fig. 5). Experimental values exceed 12S.[7] The disagreement between the two values is due to the development of three-dimensional effect at this value of Reynolds number. When the length of computational domain was doubled, the reattachment length increased only to 10S, which ruled out the possibility that the computational domain was insufficient.

$R = 5000$

Streamline plots at Reynolds number 5000 are shown in Fig. 4. Separation along the bottom channel wall is manifested in the formation of small eddies that resemble Tollmien-Schlicting waves. Streamlines on the top wall start to deflect down at $R = 125$, with first signs of separation and the formation of large-scale eddies at $R = 500$. At $R = 5000$, this forms a well-defined recirculation bubble on the top wall, starting around the end of the main recirculation zone and extending downstream a length of almost 2S. The formation of these eddies is due to the unfavorable pressure gradient associated with the expansion of the channel. Within the separating shear layer, smaller eddies appear at high Reynolds numbers around the point of separation. The presence of these eddies explains the sudden increase in the turbulent kinetic energy, especially near the bottom wall (as seen in Fig. 6). A counter-rotating eddy appears at the lower left corner of the step. Figure 4 in-

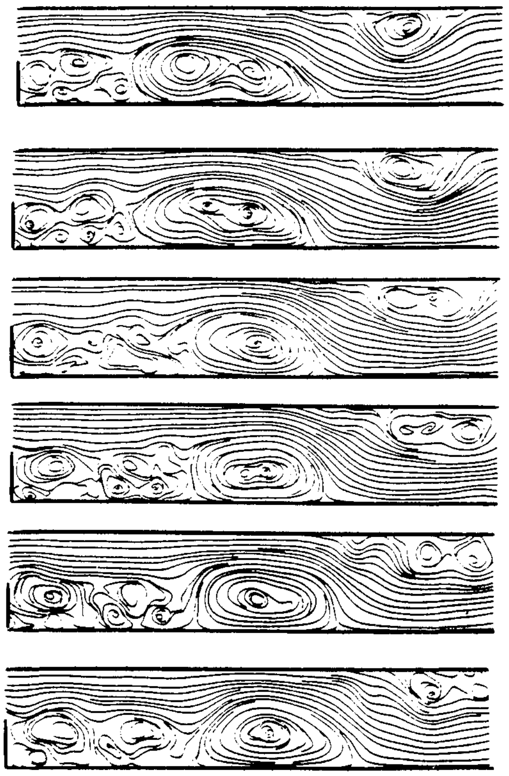

Fig. 4 Recirculation zone structure at early stages of turbulence, $R = 5000$.

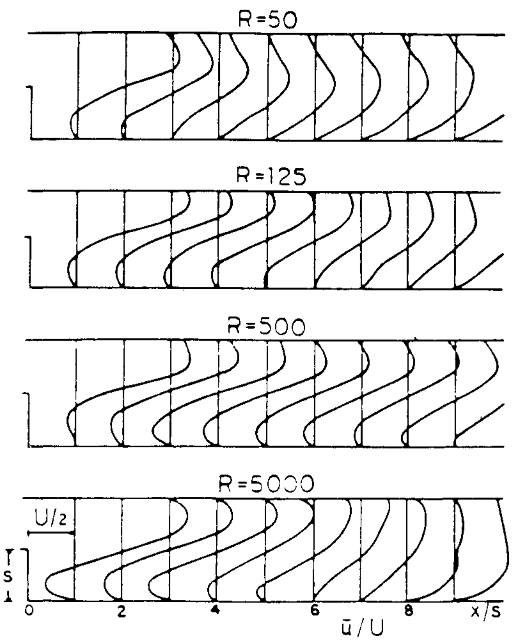

Fig. 5 Average streamwise velocity profiles for Reynolds number $R = 50$, 125, 500 and 5000. Broken lines at $y > 1.8$ indicate that this part is obtained by extrapolating the results for $y < 1.8$.

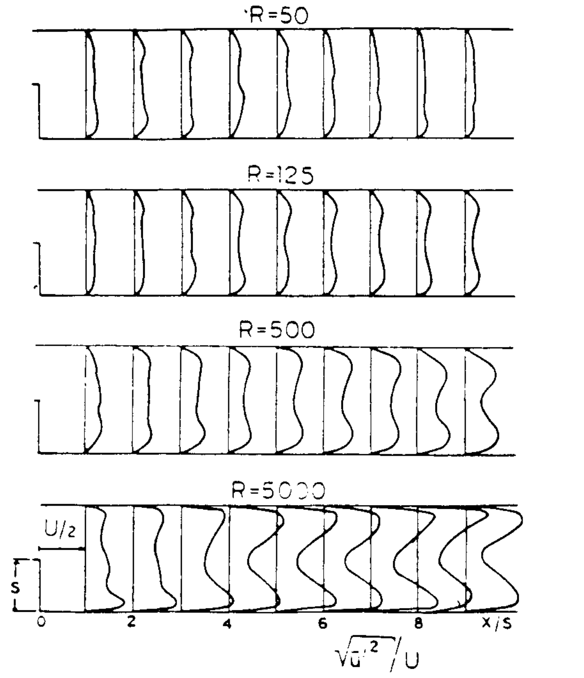

Fig. 6 Turbulent kinetic energy in the streamwise direction, $\sqrt{u'^2}$.

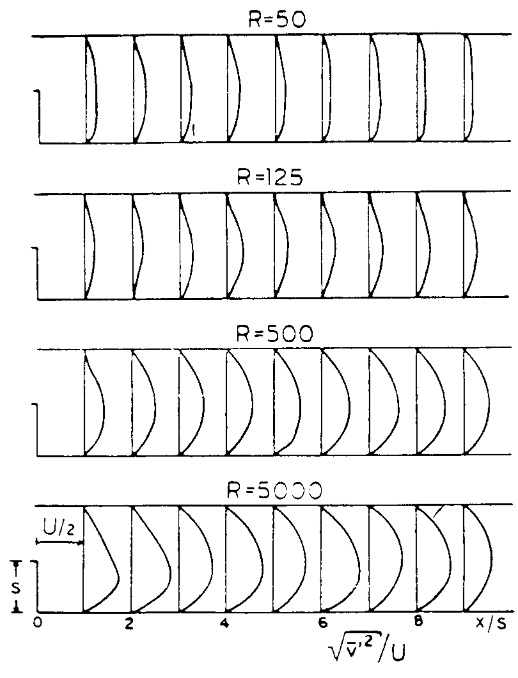

Fig. 7 Turbulent kinetic energy in the cross-stream direction.

dicates that the rate of growth of the shear layer increases over its value at lower Reynolds numbers.

Armaly et al.[7] detected a recirculation zone on the opposite wall and measured its center and size at various Reynolds numbers. Moss et al.[11] report experimental data that show the counter-rotating eddy at the step corner. Reattachment length, computed from our streamline plots in Fig. 4 and average velocity profiles in Fig. 5, is 6S, in agreement with the experimental results of , e.g., Eaton and Johnson.[12] The experimental scatter in the attachment length, as measured by different investigators, is smaller at the turbulent regime, and our numerical results fall within this narrower range of scatter. We should emphasize that, because of the limitation of the numerical simulation, the results can only capture the two-dimensional aspects of turbulent motion, i.e., the amplification of perturbations and the formation of small-scale eddies.

Velocity Statistics

Figure 5 shows streamwise velocity profiles obtained by averaging over 500 time steps between time steps 501 and 1000. Although the no-slip condition was satisfied on all walls, including the top wall, the velocity on the grid used for display was computed up to $y \leq 1.8$. The profiles reveal that 1) the reattachment length, measured by the point at which the

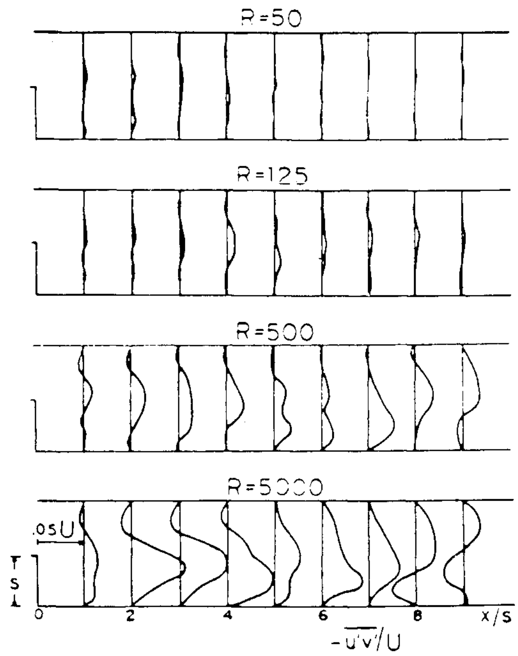

Fig. 8 Turbulent shear stress $-u'v'$ for the four values of Reynolds number considered in Figs. 5–7.

velocity reverses its sign close to the wall, increases with Reynolds number to a maximum value of about 9S at $R = 500$, then decreases to 6S at $R = 5000$; and 2) the maximum value of the recirculation velocity increases at higher Reynolds number, and the area containing fluid that moves with negative velocity becomes wider. Figures 6–8 show the average values of the second moments of velocity and their cross-correlation. Results indicate that 1) the values of $\overline{v'v'}$ and $-\overline{u'v'}$ increase with R, locally reaching a maximum around the boundary of the recirculation zone, i.e., at the shear layer, where the momentum flux across the layer is transported by the passing eddies; 2) at transition and beyond, $R > 500$, the gradient of the turbulent shear stress reaches zero on the wall at the reattachment point section; 3) the maximum computed value of $\sqrt{\overline{u'u'}} = 0.25$ occurs at $R = 5000$, whereas Dust and Tropea[10] show a wide scatter of experimental data at that range, with a mean value of about 0.2, and Atkins et al.[13] present experimental results that exhibit the same double-peaked profiles of Fig. 6 close to the step; and 4) the maximum values of $-\overline{u'v'}$ reach 0.05 and stay almost unchanged until the reattachment point, whereas experimental results[13] show a maximum of about 0.02.

Conclusions

The structure of the recirculation zone forming behind a rearward-facing step in a channel is computed for Reynolds number in the 50–5000 range, using the random vortex method. Results are analyzed in terms of instantaneous streamlines, average velocity profiles, and turbulence statistics. Four distinct flow regimes are identified. At very low Reynolds numbers, the flow is viscous, and the recirculation zone is formed of one stationary eddy at the corner of the step. As the Reynolds number increases, an eddy may detach and decay as it moves downstream. At moderate values, the flow reaches a state of transition, with eddy shedding at the step. At high Reynolds numbers, the flow exhibits turbulent behavior with a continuous process of eddy formation and pairing. The recirculation zone length increases with Reynolds number, reaching a maximum at transition and then decay to a shorter length at the turbulent range. Turbulence statistics show a rise at transition.

Acknowledgments

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Experimental Evaluation of Approximations for $\overline{w^2}$ and $\overline{vw^2}$

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Introduction

ASSUMPTIONS involved in the assessment of any physical situation should be reduced to a minimum in order to increase knowledge of a subject. The purpose of this Note is to evaluate assumptions that are usually made for some moments of turbulence fluctuating velocity. One of these moments requires additional time to measure, whereas the other is difficult to measure and is time consuming. The

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